An Algorithmic Theory of Markets and Their Application to Decentralized Markets

Denizalp Goktas
[1]Brown University
115 Waterman St, Providence, Rhode Island 02906
denizalp_goktas@brown.edu

Abstract

Broadly speaking, I hope to dedicate my PhD to improving our understanding of algorithmic economics with the ultimate goal of building welfare improving decentralized technology for markets. In the following pages, I describe how my past work has built on the existing literature to get closer to the goal of creating such technologies, and describe what research paths this work opens up for the rest of my PhD. I believe that my research has the potential to provide algorithmic solutions to problems in machine learning, optimization, and game theory, and can be used to improve the efficiency of online marketplaces.

Related and Past Work

Over the last two centuries, mathematical economists have dedicated a great deal of effort to constructing market models representative of real-world markets. This corpus of work, which culminated in the seminal work of Arrow and Debreu (1954) proving the existence of equilibrium prices in a very general setting, has come to be known as general equilibrium theory, and makes up much of the foundations of modern mainstream economics. This line of research focused mostly on the mathematical aspects of equilibrium prices, while questions regarding the algorithmic aspects, e.g., how do markets arrive at equilibrium, were left mostly unanswered.

These questions were brought back into the mainstream in 2002 (Devanur et al.), this time by computer scientists facing a series of new algorithmic problems with the proliferation of online marketplaces. These computer scientists' line of work has aimed to 1) devise efficient algorithms to compute equilibrium prices, and 2) discover decentralized and interpretable algorithms which would explain how real world markets behave, a goal which many economists abandoned, following a series of negative results. (Arrow and Kehoe 1994).

It is upon this foundation that I hope to pursue the twin goals of my research: 1) improving our understanding of games and markets by discovering decentralized algorithms that converge to economic and game-theoretic equilibria, and 2) applying these discoveries to build efficient decentralized markets.

Past Work Although the corpus of work at the intersection of economics and computer science has been steadily growing, the algorithmic progresses in game theory have often been disconnected from the economics literature. My work bridges this gap by using tools from the economics literature to improve our understanding of the algorithmic aspects of markets. For instance, using tools from consumer-theory, we have recently provided a framework to unify existing convex programs to compute market equilibria and have shown that gradient descent on these programs is equivalent to a decentralized price discovery algorithm called *tâtonnment* (Goktas, Viqueira, and Greenwald 2021).

In the last two decades, general equilibrium theory and game theory have come to be seen as intrinsically related fields of research, as seminal results in algorithmic game theory have shown that the computation of Nash equilibria and certain market equilibria are equivalent problems (Chen et al. 2009). Yet, the connection between games and existing market models has not been established in general settings. Motivated by this lack of understanding, we have shown that a large class of markets are equivalent to a type of sequential zero-sum games, i.e., a type of min-max optimization problem, and have shown an algorithmic connection between first-order methods to solve these games and decentralized price discovery algorithms (Goktas and Greenwald 2021a).

The algorithms we provided not only improve our understanding of markets, but also provide a new framework and set of tools to solve other problems of interest. For instance, very recently Dütting et al. (2021) constructed a neural network for the design of optimal auctions, whose learning objective is equivalent to solving an instance of the game model we studied. As a result, our algorithms can be used to improve the performance of neural networks in optimal auction design problems. Additionally, the algorithms we provided can be used to solve a large class of convex Robust Optimization problems (Bertsimas, Brown, and Caramanis 2011) which were previously not known to be solvable efficiently. More broadly, we believe that the type of games we solved can be used to model many problems of interest in economics, optimization, and machine learning.

Building on our work introducing the first polynomial time algorithms to solve a very general type of sequential zero-sum games (Goktas and Greenwald 2021a), we have discovered algorithms with better computational guarantees to solve such problems (Goktas and Greenwald 2021b) and have shown that these algorithms are also equivalent to novel decentralized price discovery algorithms. Even more recently, we have studied the robustness of these algorithms (Goktas and Greenwald 2021c); our findings suggest that decentralized price discovery algorithms efficiently trace market equilibria even in dynamic markets. *In sum, my research bridges the gap between economics and computer science, and in doing so provides an algorithmic theory of markets.*

New Directions

My previous work inspires a research agenda of both theoretical and applied interest. This agenda can be summarized as: 1) devise new decentralized algorithms which converge to desirable outcomes in *dynamic and stochastic* games and markets, and 2) develop open source libraries implementing decentralized price discovery algorithms to facilitate an easy adoption of these algorithms in online decentralized markets.

The theoretical results we have obtained regarding the convergence of *tâtonnement* (Goktas, Viqueira, and Greenwald 2021) currently apply to a static market model. Since real-world markets are dynamic in nature, if we would like to obtain a better understanding of how we can apply a decentralized price discovery algorithm such as *tâtonnement* in the real-world, we should investigate *tâtonnement* in dynamic market settings. We have made efforts in this direction (Goktas and Greenwald 2021c) by building on the work of Cheung, Hoefer, and Nakhe (2018) but much work remains to be done.

Finally, the results we have and continue to obtain on decentralized price discovery algorithms' convergence properties can be used to design decentralized marketplaces. With the continued rise of technologies such as Blockchain for decentralized finance (DeFi), building efficient, decentralized markets has become more important than ever. Consider the recently emerging decentralized exchange platform (DEX) technology for DeFi applications, whose main promise is to replace traditional centralized exchanges with peer-to-peer markets in the hopes of reducing transaction costs. So far, DEX platforms have aimed at building secure and computationally efficient exchange protocols between two parties, but little to no effort has been put into building decentralized protocols which guarantee efficient price discovery, i.e., efficient protocols for sellers to meet buyers without the need for an intermediary.

In fact, even though the technology used to communicate and exchange information in these markets is decentralized, little about the markets themselves is truly decentralized, as evidenced by the fact that companies such as OpenSea have emerged to serve as middlemen to connect sellers and buyers. For this reason, I hope to develop an open source library of decentralized price discovery algorithms which can allow any developer to easily implement a decentralized market over a network.

Summary

As our world becomes increasingly more automated, decentralized online marketplaces are playing an ever more important role in people's livelihoods. Creating efficient such marketplaces, however, requires overcoming difficult theoretical and practical challenges. My research aims to address these challenges, as it has the potential to provide algorithmic solutions to relevant problems in machine learning, optimization, and algorithmic game theory. I aim to contribute to the development of an algorithmic theory of markets that can be used to improve real-world markets, and hence have a positive impact on society at large.

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